Tribhuvan University

Institute of Science and Technology

2069

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Bachelor Level / First Year/ Second Semester/ Science
Computer Science and Information Technology (MTH. 155)
(Linear Algebra)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions:

(10x2=20)

Group A

1. What do you mean by linearly independent set and linearly dependent set of vectors? 2

2. Verify that $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is an eigen values of $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$. 2

3. What do you mean by consistent equations? Give suitable examples. 2

4. What do you mean by change of basis in $\mathbb{R}^n$?

5. Find the dimension of the vector space spanned by (1, 1, 0) and (o, 1, 0). 1

6. Solve the system $3x_1 + 4x_2 = 3$, $5x_1 + 6x_2 = 7$ by using the inverse of the matrix $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$. 2

7. When is a linear transformation invertable? 2

8. Find the rank of AB where

$$A = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \text{ and } B = [1\ 4\ 5]$$ 2

9. Define Kernel and image of linear transformation. 1

10. What is meant by Discrete dynamical system? Give suitable example.

Group B

(5x4=20)

11. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T(x, y, z) = (x, y, x - 2y)$. Find a basis and dimension of (a) Ker T (b) Im T.

12. Show that the following sets of vectors are linearly independent: (1,1,2), (3,1,2), (0,1,4). 4

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13. Find the matrix representation of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + 2y)$ relative to the standard basis.

14. Is the set of vectors $\{(1, 0, 1), (0, 1, 0), (-1, 0, 1)\}$ orthogonal? Obtain the corresponding orthonormal set in $\mathbb{R}^3$.

15. Let the four vertices O(0, 0), A(1, 0), B(0, 1) and C(1, 1) of a unit square be represented by 2×4 matrix : $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. Inverstigate and interpret geometrically the effect of pre-multiplication f this matrix by the 2×2 matrix: $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$.

OR

State and prove orthogonality property for any two non-zero vectors in $\mathbb{R}^n$.

Group C

(5x8=40)

16. Find a matrix A whose inverse in

$$A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

17. Test the consistency and slove:

$x + y + z = 4$

$x + 2y + 2z = 2$

$2x + 2y + z = 5$

OR

Verify Cayley Hamilton theorem for a matrix $A = \begin{bmatrix} 6 & 2 & -1 \\ -6 & -1 & 2 \\ 7 & 2 & -2 \end{bmatrix}$.

18. The set of matrices of the form

$$\begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$$

is a subspace of the vector space of 3×3 matrices. Verify it.

W = m. Let $\{e_1, e_2, \ldots, e_n\}$ be a basis of V and $\{f_1, f_2, \ldots, f_m\}$ be a basis of W. Then, prove that each linear transformation T: V → W can be represented by an m × n matrix A with elements from F such that

$$Y = AX$$

where $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and $Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$

are column matrices of coordinates of v ∈ V relative to its basis and coordinates of w ∈ W relative to its basis, respectively.

OR

Compute the multiplication of partitioned matrices for 8

$$A = \left[\begin{array}{ccc:cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hdashline 0 & 4 & -2 & 7 & -1 \end{array}\right] \quad \text{and} \quad B = \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hdashline -1 & 3 \\ 5 & 2 \end{array}\right].$$

20. *Find the equation* $y = a_0 + a_1 x$ *for the least squares line that best fits the data points* (2, 1), (5, 2), (7, 3), (8, 3). 8